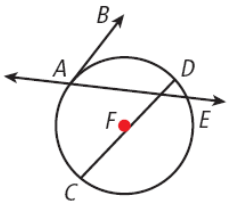
Attendance Problems.

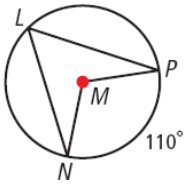
1. Identify each line or segment in relation to $\odot F$.



Find each measure.

2. $m\measuredangle NMP$

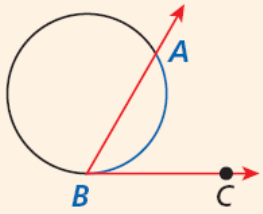
3. $m\measuredangle NLP$



- I can find the measures of angles formed by lines that intersect circles.
- I can use angle measures to solve problems.

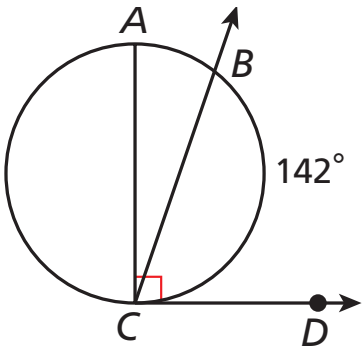
Common Core: **CC.9-12.G.C.2** Identify and describe relationships among inscribed angles, radii, and chords.

Theorem 11-5-1

THEOREM	HYPOTHESIS	CONCLUSION
If a tangent and a secant (or chord) intersect on a circle at the point of tangency, then the measure of the angle formed is half the measure of its intercepted arc.	 Tangent $\overrightarrow{BC}$ and secant $\overrightarrow{BA}$ intersect at B.	$m\angle ABC = \frac{1}{2}m\overset{\frown}{AB}$

1 Using Tangent-Secant and Tangent-Chord Angles

Find each measure.



A $m\angle BCD$

$$m\angle BCD = \frac{1}{2}m\overset{\frown}{BC}$$

$$m\angle BCD = \frac{1}{2}(142°)$$

$$= 71°$$

B $m\overset{\frown}{ABC}$

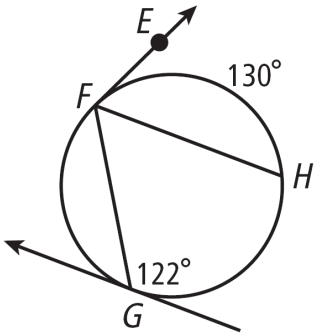
$$m\angle ACD = \frac{1}{2}m\overset{\frown}{ABC}$$

$$90° = \frac{1}{2}m\overset{\frown}{ABC}$$

$$180° = m\overset{\frown}{ABC}$$

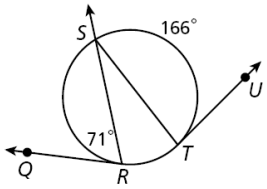
Example 1. Find each measure.

A. $m\angle EFH$ **B.** $m\overset{\frown}{GF}$

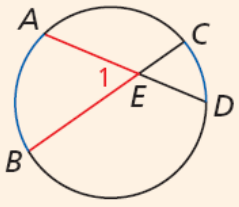


<u>Guided Practice.</u> Find each measure.

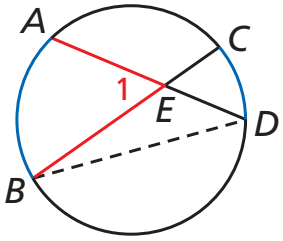
4. $m\angle STU$ **5.** $m\overset{\frown}{SR}$



Theorem 11-5-2

THEOREM	HYPOTHESIS	CONCLUSION
If two secants or chords intersect in the interior of a circle, then the measure of each angle formed is half the sum of the measures of its intercepted arcs.	 Chords $\overline{AD}$ and $\overline{BC}$ intersect at E.	$m\angle 1 = \frac{1}{2}\left(m\overset{\frown}{AB} + m\overset{\frown}{CD}\right)$

Theorem 12-5-2



Given: $\overline{AD}$ and $\overline{BC}$ intersect at E.

Prove: $m\angle 1 = \frac{1}{2}\left(m\overset{\frown}{AB} + m\overset{\frown}{CD}\right)$

Proof:

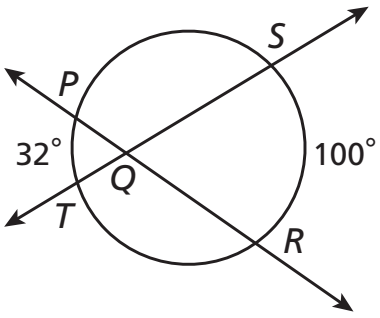
Statements	Reasons
1. $\overline{AD}$ and $\overline{BC}$ intersect at E.	**1.** Given
2. Draw $\overline{BD}$.	**2.** Two pts. determine a line.
3. $m\angle 1 = m\angle EDB + m\angle EBD$	**3.** Ext. $\angle$ Thm.
4. $m\angle EDB = \frac{1}{2}m\overset{\frown}{AB}$, $m\angle EBD = \frac{1}{2}m\overset{\frown}{CD}$	**4.** Inscribed $\angle$ Thm.
5. $m\angle 1 = \frac{1}{2}m\overset{\frown}{AB} + \frac{1}{2}m\overset{\frown}{CD}$	**5.** Subst.
6. $m\angle 1 = \frac{1}{2}\left(m\overset{\frown}{AB} + m\overset{\frown}{CD}\right)$	**6.** Distrib. Prop.

2 Finding Angle Measures Inside a Circle

Find each angle measure.

$m\angle SQR$

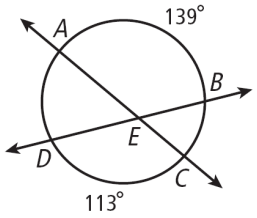
$$m\angle SQR = \frac{1}{2}\left(m\overset{\frown}{PT} + m\overset{\frown}{SR}\right)$$
$$= \frac{1}{2}(32° + 100°)$$
$$= \frac{1}{2}(132°)$$
$$= 66°$$



Question: Why was the chord upset with the tangent after their race?
Answer: They came in secant!

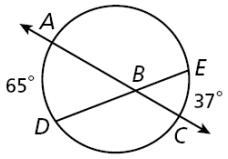
"Don't pray when it rains if you don't pray when the sun shines."—*Baseball Player, Satchel Paige*

Example 2. Find $m\measuredangle AEB$

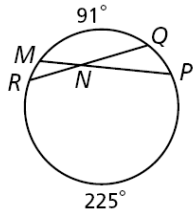


<u>Guided Practice.</u> Find each measure.

6. $m\measuredangle ABD$

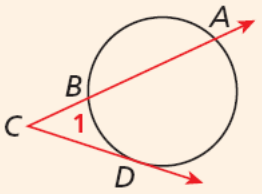


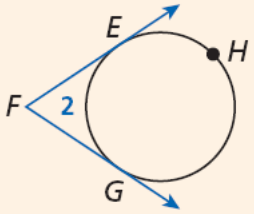
7. $m\measuredangle RNM$

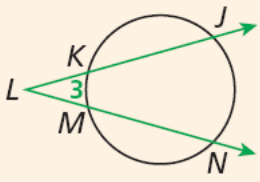


Theorem 11-5-3

If a **tangent and a secant**, **two tangents**, or **two secants** intersect in the exterior of a circle, then the measure of the angle formed is half the difference of the measures of its intercepted arcs.





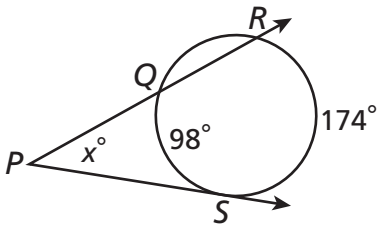


$m\angle 1 = \frac{1}{2}\left(m\widehat{AD} - m\widehat{BD}\right)$ $m\angle 2 = \frac{1}{2}\left(m\widehat{EHG} - m\widehat{EG}\right)$ $m\angle 3 = \frac{1}{2}\left(m\widehat{JN} - m\widehat{KM}\right)$

3 Finding Measures Using Tangents and Secants

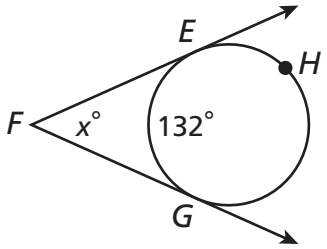
Find the value of x.

A



$$x = \frac{1}{2}\left(m\widehat{RS} - m\widehat{QS}\right)$$
$$= \frac{1}{2}(174° - 98°)$$
$$= 38°$$

B

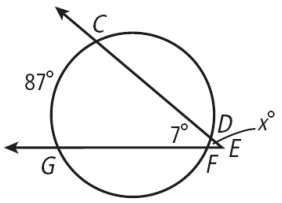


$$x = \frac{1}{2}\left(m\widehat{EHG} - m\widehat{EG}\right)$$
$$= \frac{1}{2}(228° - 132°)$$
$$= 48°$$

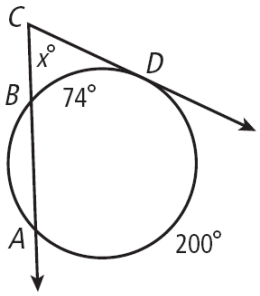
"Yep, I see it. Right there on the tip - 'hypotenuse.'"

Example 3. Find the value of x.

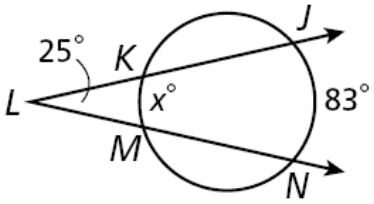
A.



B.



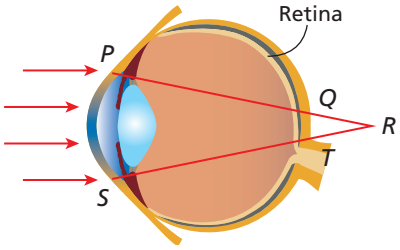
8. Guided Practice.



12-5 Angle Relationships in Circles (p 835) 17, 19, 21, 22, 23, 25.

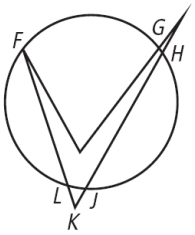
4 ***Biology Application***

When a person is farsighted, light rays enter the eye and are focused behind the retina. In the eye shown, light rays converge at *R*. If $m\widehat{PS} = 60°$ and $m\widehat{QT} = 14°$, what is $m\angle PRS$?

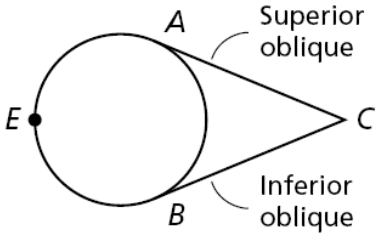


$$m\angle PRS = \frac{1}{2}\left(m\widehat{PS} - m\widehat{QT}\right)$$
$$= \frac{1}{2}(60° - 14°)$$
$$= \frac{1}{2}(46°) = 23°$$

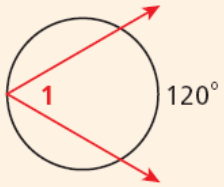
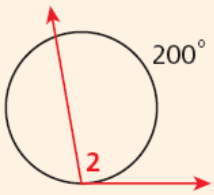
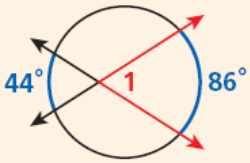
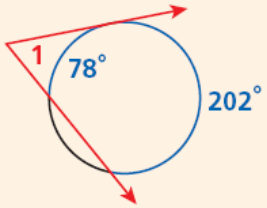
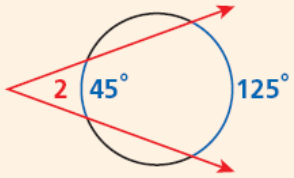
Example 4. In the company logo shown, $m\widehat{FH} = 108°$ & $m\widehat{LJ} = 12°$. What is $m\measuredangle FKH$?

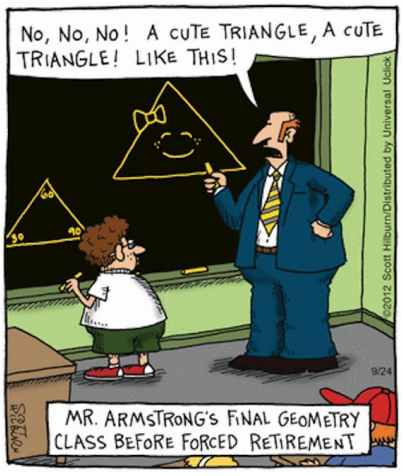


9. Guided Practice. Two of the six muscles that control eye movement are attached to the eyeball and intersect behind the eye. If $m\widehat{AEB} = 225°$,, what is $m\measuredangle ACB$?

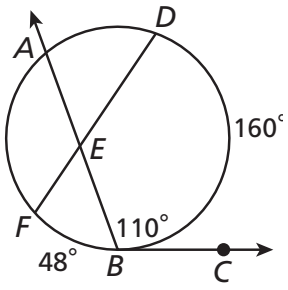


Angle Relationships in Circles

VERTEX OF THE ANGLE	MEASURE OF ANGLE	DIAGRAMS
On a circle	Half the measure of its intercepted arc	 $m\angle 1 = 60°$  $m\angle 2 = 100°$
Inside a circle	Half the sum of the measures of its intercepted arcs	 $m\angle 1 = \frac{1}{2}(44° + 86°)$ $= 65°$
Outside a circle	Half the difference of the measures of its intercepted arcs	 $m\angle 1 = \frac{1}{2}(202° - 78°)$ $= 62°$  $m\angle 2 = \frac{1}{2}(125° - 45°)$ $= 40°$



5 Finding Arc Measures



Find $m\widehat{AF}$.

Step 1 Find $m\widehat{ADB}$.

$m\angle ABC = \frac{1}{2}m\widehat{ADB}$ *If a tangent and secant intersect on a ⊙ at the pt. of tangency, then the measure of the ∠ formed is half the measure of its intercepted arc.*

$110° = \frac{1}{2}m\widehat{ADB}$ *Substitute 110 for m∠ABC.*

$m\widehat{ADB} = 220°$ *Mult. both sides by 2.*

Step 2 Find $m\widehat{AD}$.

$m\widehat{ADB} = m\widehat{AD} + m\widehat{DB}$ *Arc Add. Post.*

$220° = m\widehat{AD} + 160°$ *Substitute.*

$m\widehat{AD} = 60°$ *Subtract 160 from both sides.*

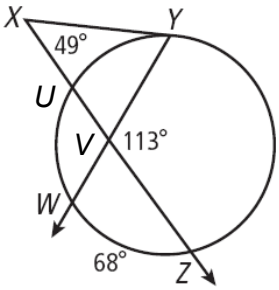
Step 3 Find $m\widehat{AF}$.

$m\widehat{AF} = 360° - (m\widehat{AD} + m\widehat{DB} + m\widehat{BF})$ *Def. of a ⊙*

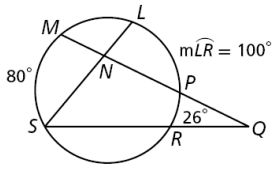
$= 360° - (60° + 160° + 48°)$ *Substitute.*

$= 92°$ *Simplify.*

Example 5. Find $m\widehat{YZ}$.



10. <u>Guided Practice</u>. Find $m\widehat{LP}$



12-5 Angle Relationships in Circles (p 835) 17, 19, 21, 22, 23, 25-29, 30, 31-33, 37, 38, 40-43.

Question: Why was the chord upset with the tangent after their race?
Answer: They came in secant!